# The UNNS Vector Protocol (UVP):
# A Formalization of Recursive Vectors in the UNNS Substrate

UNNS Research Notes

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**Abstract**

The Unbounded Nested Number Sequence (UNNS) substrate encodes recursive systems through operators acting on nests. To integrate UNNS into broader mathematics and physics, we develop the *UNNS Vector Protocol* (UVP), which formalizes nests as elements of layered vector spaces. The UVP provides linear algebraic structure, operator actions, and bridges to topological field theory and gauge physics.

## Contents



## 1 Motivation

While UNNS operators capture recursion, they require a unifying linear framework for composition, projection, and superposition. The UVP serves this role, mapping nests into vectors, equipping the system with linear structure, and providing compatibility with tensor calculus and field theories.

## 2 Vectorization of Nests

**Definition 2.1** (Nest Vectorization)**.** *Let $\mathcal{N} = (a_0, a_1, a_2, \dots)$ be a UNNS nest. The* nest vectorization map $V$ *sends $\mathcal{N}$ into a formal vector*

$$V(\mathcal{N}) = \sum_{k \geq 0} a_k e_k \in \mathbb{V},$$

*where $\{e_k\}_{k=0}^{\infty}$ is a canonical basis of the UNNS vector space $\mathbb{V}$.*

**Remark 2.2.** *This construction embeds recursive sequences into an infinite-dimensional vector space, allowing linear operations such as superposition and inner products.*

## 3 The UNNS Vector Protocol

### 3.1 Protocol Rules

The UVP defines the following principles:

1. **Embedding:** Every admissible nest maps to a vector in $\mathbb{V}$.
2. **Operator Action:** Each UNNS operator acts linearly (or piecewise linearly) on $\mathbb{V}$.
3. **Stability:** Repair and evaluation guarantee boundedness in appropriate norms.
4. **Projection:** Projection $\Pi$ reduces $\mathbb{V}$ to finite-dimensional subspaces for computation or perception.

### 3.2 Operator Representations

**Proposition 3.1.** *The core operators act as follows on $V(\mathcal{N})$:*

$$\begin{aligned}
\mathcal{I}: &\quad V(\mathcal{N}) \mapsto V(\mathcal{N}) + xe_0,\\
\mathcal{J}: &\quad V(\mathcal{N}) \mapsto V(\mathcal{N}) \oplus V(\mathcal{M}),\\
\mathcal{R}: &\quad V(\mathcal{N}) \mapsto P_{stable}(V(\mathcal{N})),\\
\mathcal{T}: &\quad V(\mathcal{N}) \mapsto \Phi(V(\mathcal{N})),
\end{aligned}$$

*where $P_{stable}$ is a stability projection and $\Phi$ a perceptual transform.*

**Remark 3.2.** *Other operators (branching, merging, decomposing, etc.) have analogous linear or bilinear forms.*

## 4 Theorems on UVP Structure

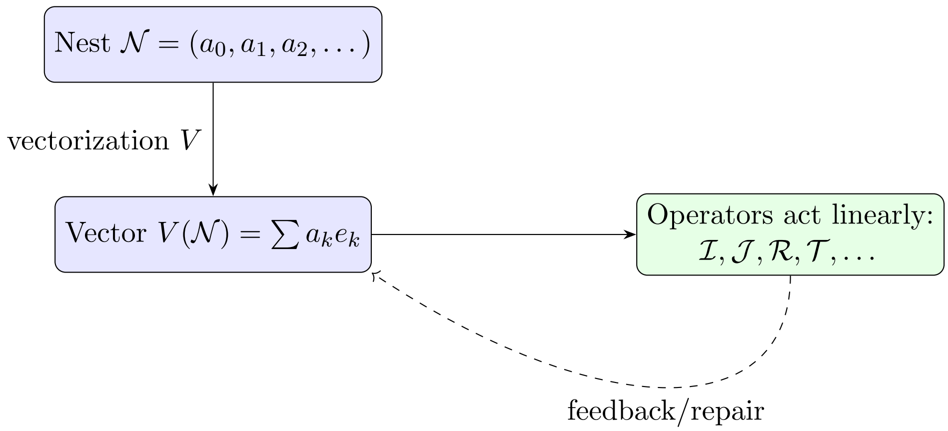
**Theorem 4.1** (Linearity of Embedding)**.** *The vectorization map $V$ is linear: for any two nests $\mathcal{N}_1, \mathcal{N}_2$,*

$$V(\mathcal{N}_1 + \mathcal{N}_2) = V(\mathcal{N}_1) + V(\mathcal{N}_2).$$

*Proof.* Follows from coordinatewise addition of sequence entries and linearity of basis vectors. □

**Lemma 4.2** (Stability norm)**.** *There exists a norm $\|\cdot\|_{\mathbb{V}}$ on $\mathbb{V}$ such that all repaired nests satisfy $\|V(\mathcal{N})\|_{\mathbb{V}} < \infty$.*

# 5 Diagrammatic Overview




# 6 Applications

## 6.1 Mathematics

- Provides a basis for *UNNS linear algebra*.
- Embeds recurrence relations into operator theory.

## 6.2 Physics

- Enables gauge-like interpretations of operator actions.
- Serves as a discrete analog of vector/tensor fields.

## 6.3 Computation

- Facilitates efficient encoding of recursion as matrix operations.
- Provides a path toward UNNS numerical solvers.

# 7 Conclusion

The UNNS Vector Protocol provides the linear backbone for UNNS. It integrates recursion with vector space structure, ensures operator actions are well-defined, and enables bridges to physics, computation, and higher mathematics.